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Question Paper Code : 21773

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

(Use of Statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The cummulative distribution function of the random variable X is given by

$$F_X(x) = \begin{cases} 0; \ x < 0 \\ x + \frac{1}{2}; 0 \le x \le \frac{1}{2} \\ 1 \quad ; \ x > \frac{1}{2} \end{cases}, \text{ compute } P\left[X > \frac{1}{4}\right].$$

2. Find the variance of the discrete random variable X with the probability mass function $P_X(x) = \begin{cases} \frac{1}{3} & x = 0 \\ \frac{1}{2} & x = 2 \end{cases}$.

3. If X, Y denote the deviation of variance from the arithmetic mean and if $P = 0.5$, $\Sigma XY = 120$, $\sigma_y = 8$, $\Sigma X^2 = 90$. Find n, number of times.

4. For $\lambda > 0$, let $F(x,y) = \begin{cases} 1 - \lambda e^{-\lambda(x+y)}, & if \ x > 0, y > 0 \\ 0 & , otherwise \end{cases}$ check whether F can be the joint probability distribution function of two random variables X and Y.

5. Define first-order stationary processes.

6. Suppose that $X(t)$ is a Gaussian process with $\mu_X = 2, R_{XX} = (\tau) = 5e^{-0.2|\tau|}$, find the probability that $X(4) \leq 1$.

7. Prove that the auto correlation function is an even function of τ.

8. State Wiener-Khinchine theorem.

9. Check whether the system $Y(t) = X^3(t)$ is linear.

10. Compare band-limited white noise with ideal low-pass filtered white noise.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The members of a girl scout troop are selling cookies from house to house in town. The probability that they sell a set of cookies at any house they visit is 0.4.

(1) If they visit 8 houses in one evening, what is the probability that they sold cookies to exactly five of these houses?

(2) If they visited 8 houses in one evening, what is the expected number of sets of cookies they sold?

(3) What is the probability that they sold their set of cookies atmost in the sixth house they visited? (8)

(ii) Suppose X has an exponential distribution with mean equal to 10. Find the value of x such that $P(x < x) = 0.95$. (8)

Or

(b) (i) If the moments of a random variable X are defined by $E(X^r) = 0.6$, $r = 1, 2....$ Show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$ and $P(X \geq 2) = 0$. (8)

(ii) Find the probability density function of the random variable $y = x^2$ where X is the standard normal variate. (8)

12. (a) (i) The joint PMF of two random variables X and Y is given by $P_{XY}(x, y) = \begin{cases} K(2x + y) & x = 1, 2; y = 1, 2 \\ 0 & otherwise \end{cases}$, where K is a constant

(1) Find K

(2) Find the marginal PMFs of X and Y. (8)

(ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose PDF is given by $f_X(x)=\begin{cases}\frac{1}{3} & 1\le x\le 4\\ 0, \textit{otherwise}\end{cases}$. Find the probability that S_n lies in the range $108 \le S_n \le 126$. (8)

Or

(b) (i) Two random variables X and Y are related as $Y = 4X + 9$. Find the correlation coefficient between X and Y. (8)

(ii) If the density function is defined by $f(x,y)=\frac{y}{(1+x)^4}e^{\frac{-y}{1+x}}$, $x \ge 0, y \ge 0$ then obtain the regression equation of Y on X for the distribution. (8)

13. (a) (i) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide sense stationary where A and ω_0 are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) For the random process $X(t) = A\cos\omega t + B\sin\omega t$ where A and B are random variables with $E(A) = E(B) = 0$, $E(A^2) = E(B^2) > 0$ and $E(AB) = 0$. Prove that the process is mean Ergodic. (8)

Or

(b) (i) Two boys B_1, B_2 and 2 girls G_1, G_2 are throwing a ball from one to another. Each boy throws the ball to other boy with probability 1/2 and to each girl with probability 1/4. On the other hand, each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run, how does each receive the ball? (8)

(ii) If $\{X(t)\}$ is a Poisson process, then prove that correlation coefficient between $X(t)$ and $X(t+s)$ is $\sqrt{\frac{t}{t+s}}$. (8)

14. (a) (i) Find the spectral density of a WSS random process $\{X(t)\}$ whose auto correlation function is $e^{\frac{-\alpha^2 t^2}{2}}$. (8)

(ii) Find the auto correlation function of the WSS process $\{X(t)\}$ whose spectral density is given by $S(\omega) = \frac{1}{(1+\omega^2)^2}$. (8)

Or

(b) (i) The cross-power spectrum of real random process $\{X(t)\}$ and $\{Y(t)\}$ is given by $S_{XY}(\omega)=\begin{cases} a+jb\omega, & |\omega|<1 \\ 0 & elsewhere \end{cases}$. Find the cross-correlation function. (8)

(ii) Determine the cross correlation function corresponding to the cross-power density spectrum $S_{XY}(\omega)=\dfrac{8}{(\alpha+j\omega)^3}$, where $\alpha>0$ is a constant. (8)

15. (a) (i) If the output of the input $X(t)$ is defined as $Y(t)=\dfrac{1}{T}\int_{t-T}^{T} X(s)\,ds$, prove that $X(t)$ and $Y(t)$ are related by means of convolution integral. Find the unit impulse response of the system. (8)

(ii) A circuit has an impulse response given by $h(t)=\begin{cases} \dfrac{1}{T}, 0\le t\le T \\ 0 \quad otherwise \end{cases}$. Evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$. (8)

Or

(b) (i) Given that $y(t)=\dfrac{1}{2\in}\int_{t-\in}^{1+\in} X(\alpha)\,d\alpha$ where $\{Y(t)\}$is a WSS process, prove that $S_{YY}(\omega)=\dfrac{\sin^2\in\omega}{\in^2\omega^2}S_{XX}(\omega)$. Find the output auto correlation function. (8)

(ii) A linear time invariant system has an impulse response $h(t)=e^{-\beta t}\omega(t)$. Find the output auto correlation function $R_{YY}(\tau)$ corresponding to an input $X(t)$. (8)
